TRINITY COLLEGE

Year 12 Maths Applications
Test 2 2022

Section 1 Calculator Free
Sequences and Series

STUDENT'S NAME SOLUTIONS

DATE: Wednesday 30th March **TIME:** 20 minutes **MARKS:** 21

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (3 marks)

For each of the following sequences below state whether it is arithmetic, geometric or neither,

(a) $20, \ -40, \ 80, \ -160$ [1]

GP ✓

(b) $8, \ 19, \ 29, \ 39$ [1]

NEITHER ✓

(c) $4, \ 9, \ 16, \ 25$ [1]

NEITHER ✓

3

2. (2 marks)

A sequence with a first term 20 000 has a sequential decrease of 15%.

Determine the recursive rule for T_{n+1} in terms of T_n

$T_{n+1} = 0{\cdot}85\, T_n$, ✓

2

$T_1 = 20000$ ✓

3. (3 marks)

State the first three terms of the following sequence

$T_n \quad = \quad -2^n \times T_{n-1} \;\; + 3 \qquad\qquad T_1 \;=\; 2$

$T_1 = \underline{\underline{2}}$

$T_2 = -2^2 \times 2 + 3$

$= -4 \times 2 + 3$ ✓

$= \underline{\underline{-5}}$ ✓

$T_3 = -2^n \times -5 + 3$

$= -2^3 \times -5 + 3$

$= -8 \times -5 + 3$

$= \underline{\underline{43}}$ ✓

3

5

4. (6 marks)

A culture of bacteria in a beaker doubles every hour for 14 hours at which point the beaker is at full capacity. The first 5 hours (n) are listed in the table below with population P_n

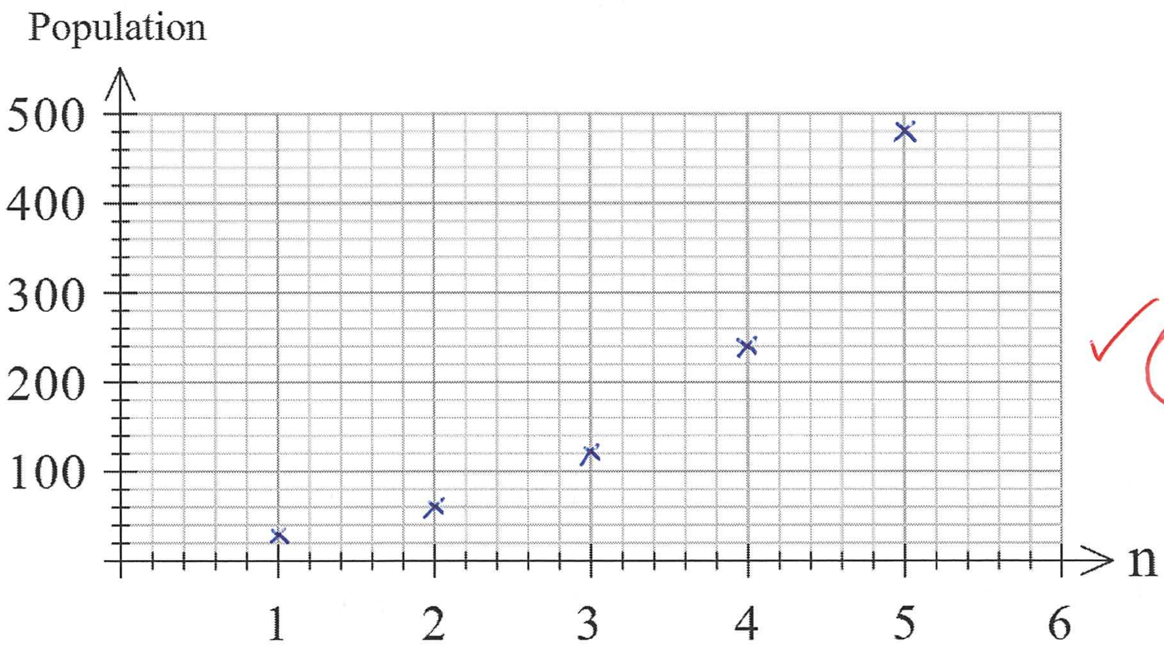
n	1	2	3	4	5
P_n	30	60	120	240	480

(a) Define the sequence recursively using P_n [2]

$P_n = 2P_{n-1}$ ✓, $P_1 = 30$ ✓

2

(b) Plot the sequence for $n = 1$ to 5 on the set of axes below [1]



✓ 1

(c) Calculate the general rule for this sequence [2]

$T_n = ar^{n-1}$

$\therefore P_n = 30 \times 2^{n-1}$ ✓ ✓ 2

OR $P_n = 15 \times 2^n$

(d) After how many hours is the beaker exactly half full? [1]

13 hours ✓ 1

/6

5. (3 marks)

For a geometric progression, calculate the general rule (n), in which the 2^{nd} term is 6 and the 4^{th} term is 54.

$T_2 = ar$

$\therefore 6 = ar$

$T_4 = ar^3$

$\therefore 54 = ar^3$

$\therefore r^2 = 9$

$r = \pm 3$ ✓

$T_n = 2 \times 3^{n-1}$ ✓ 3

$T_n = 2 \times (-3)^{n-1}$ ✓

6. (4 marks)

Sequence A has terms $A_1,\ A_2,\ A_3, \ldots$ and sequence B has terms $B_1,\ B_2,\ B_3 \ldots$.

Given that $A_1 = 8$, and $A_n = A_{n-1} + 3$, and $B_1 = 120$, and $B_n = 0.5B_{n-1}$,

determine the first three terms of the sequence of C given that $C_n = B_n - A_n$

$A_1 = 8$

$A_2 = 11$

$A_3 = 14$ ✓

$B_1 = 120$

$B_2 = 60$

$B_3 = 30$ ✓

4

$\therefore C_1 = 120 - 8 = \underline{112}$

$C_2 = 60 - 11 = \underline{49}$ ✓✓

$C_3 = 30 - 14 = \underline{16}$

-1m per mark

7

TRINITY COLLEGE

IN NOMINE DOMINI

Year 12 Maths Applications
Test 2 2022

Section 2 Calculator Assumed

Sequences and Series

STUDENT'S NAME SOLUTIONS

DATE: Wednesday 30th March **TIME:** 20 minutes **MARKS**: 19

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

7. (5 marks)

(a) A geometric sequence begins at one million and has a common ratio of three quarters. Calculate T_9. [1]

100 113 ✓ 1

(b) Kelly Swamp is expanding by 11% each year. If the Swamp was initially 34 000 kL, how many whole years will it be until the swamp is double its initial size? [2]

$68000 = 34000 \times 1.11^{n-1}$

$n = 7.6$ ✓

$\therefore$ 8 yrs ✓ 2

(c) A sequence is defined as $Tn = 3T_{n-1} + 9,\ T_2 = 3$. What are the first three terms? [2]

$-2,\ 3,\ 18$

✓ ✓ 2

 5

8. (4 marks)

Luke wants to buy a van and decides to open an account and deposits an initial \$4 500 into the account, at the end of the first week he deposits \$40 into the account and continues to do this at the end of every week. Assume his account does not accrue interest or fees.

(a) Determine a rule for the n^{th} term, where n is the amount in the account at the end of the n^{th} week. [2]

$T_n = 4500 + 40n$ 2

(b) Luke's aunt Dolly has a van she will sell to him for \$5 400. How many weeks will he need to save to buy the van off his aunt? [2]

$n = 22.5$ $\therefore$ 23 WEEKS 2

9. (5 marks)

Every year Ms Fitt swims in the TC Staff PCG Swimming Relay and always keeps track of the time she takes to swim the 50m freestyle. In 2020 she completed the race in 30.00 sec, 2021 in 33.00 sec, and 2022 in 36.30 sec. Ms Fitt accepts that she is getting slower and her decrease in speed was a recursive sequence.

(a) State, with clear reasoning if the sequence is geometric, arithmetic or neither. [2]

$\frac{33}{30} = \frac{36.3}{33} = 1.1$ $\therefore$ GP 2

(b) What time would Ms Fitt expect to complete the 50m freestyle event in 2023? [1]

39.93 sec 1

(c) The very competitive TC staff told Ms Fitt that if she can't complete the race in one minute or less she should "retire" from the team. Assuming that Ms Fitt remains working at Trinity, what year will be her last competing in the 50m freestyle event? [2]

$60 = 30 \times 1.1^{n-1}$

$n = 8.27$

$\therefore$ last year is 2027 2

9

10. (5 marks)

Anna has a swimming pool that is totally full on the first day of summer (T_1). The pool holds 60 000 litres. During the day the pool evaporates 9% of its water. To counter this water loss, Anna adds 200L every day.

(a) Determine the recursive formula of number of litres of water, W_n, in the pool on n^{th} day. [2]

$$W_n = 0.91 W_{n-1} + 200 \checkmark$$

$$W_1 = 60\,000 \checkmark$$

2

(b) Calculate how many litres the pool contains on the fifth day. [1]

$$W_5 = \underline{41\,843L} \checkmark$$

1

(c) Anna wishes to maintain a steady state in the pool of 45 000 Litres. Calculate how much water needs to be added every day to achieve this. [2]

$$45\,000 = 0.91(45\,000) + x \checkmark$$

$$x = \underline{4050\ L} \checkmark$$

2